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THE EQUIVARIANT COVERING HOMOTOPY PROPERTY FOR DIFFERENTIABLE G-FIBRE BUNDLES

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Let G be a compact Lie group, and X a differentiable G-manifold. If $p: E \to X$ is a differentiable fibre bundle, and G acts differentiably on E so that each $g \in G$ operates as a bundle map, then we call p a differentiable G-fibre bundle. We show that if p is a differentiable G-fibre bundle with Lie structure group or compact fibre, then it has the equivariant covering homotopy property. This generalizes the fact that a differentiable family of actions of a compact Lie group on a compact differentiable manifold is locally trivial.

We give some basic definitions in § 1, and in § 2 show that if X is a G-manifold and $E \to X$ a differentiable fibre bundle with Lie structure group H and associated principal bundle $P \to X$, then differentiable actions of G on E as a group of bundle maps are in natural one-one correspondence with such actions on P. In § 3 we establish the equivariant covering homotopy property for differentiable G-fibre bundles with compact Lie structure group, and show that if $p: E \to X$ is a differentiable G-fibre bundle with connected semi-simple Lie structure group H, then p can be reduced to a compact subgroup of H so that G still operates as a group of bundle maps, and hence p also has the equivariant covering homotopy property. Then in § 4 we define a notion of equivariant local triviality for G-fibre bundles, which implies the equivariant covering homotopy property, and show that any differentiable G-fibre bundle with Lie structure group or compact fibre is G-locally trivial. We conclude with some remarks relating G-local triviality to the equivalence of nearby differentiable actions of a compact Lie group.

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1. Basic definitions

Let G be a topological group. A *G-space* is a Hausdorff space X together with a *continuous action* of G on X, i.e., a continuous map $(g, x) \to gx$ of $G \times X$ into X such that $g_1(g_2x) = (g_1g_2)x$ for all $g_1, g_2 \in G$, $x \in X$, and $1x = x$,

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