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THE PLATEAU PROBLEM FOR SURFACES OF PRESCRIBED MEAN CURVATURE IN A RIEMANNIAN MANIFOLD

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1. Introduction

In this work we treat the problem of finding a surface of prescribed mean curvature in a three-dimensional riemannian manifold M, with a given closed curve as boundary. That is, given a real-valued function $H(z)$ defined on M, we wish to find a mapping $z: B \to M$, B denoting the two-dimensional unit disk, which satisfies the following conditions:

(i) $z \in C^2(B) \cap C^0(\bar{B})$,
(ii) z maps ∂B homeomorphically onto Γ,
(iii) z satisfies in B the systems

$$\nabla_{z_u} z_u + \nabla_{z_v} z_v = 2H(z)^*(z_u \wedge z_v) , \tag{1.1}$$

$$\langle z_u, z_u \rangle - \langle z_v, z_v \rangle = \langle z_u, z_v \rangle = 0 . \tag{1.2}$$

Here $\langle\ ,\ \rangle$ denotes the inner product on the tangent bundle of M, ∇ the associated Levi-Civita connection, $^*P$ the tangent vector associated with a two-vector P using $\langle\ ,\ \rangle$. Let g_{ij} be the coefficients of $\langle\ ,\ \rangle$ in some coordinate system. We may write explicity

$$^*(z_u \wedge z_v)^k = \sqrt{g} \begin{vmatrix} g^{1k} & g^{2k} & g^{3k} \\ z_u^1 & z_u^2 & z_u^3 \\ z_v^1 & z_v^2 & z_v^3 \end{vmatrix} ,$$

where $g_{ij}g^{jk} = \delta_i^k$ and $g = \det(g_{ij})$. (1.2) states that z is a conformal mapping on its image (possibly with degenerate points); under that condition, (1.1) become the equations for mean curvature $H(z)$ at regular points.

The basic result of the present paper for smooth complete M may be stated as follows. Let K_0 denote an upper bound on sectional curvatures of M, and $\Phi(r)$ the mean curvature with respect to an inward normal of the geodesic sphere of radius r in the space of constant curvature K_0. Explicitly, $\Phi(r) = \sqrt{K_0}\cot(\sqrt{K_0}\, r)$. In the case $K_0 > 0$, replace Φ by any smaller function ϕ

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