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SPECIAL CONNECTIONS AND ALMOST FOLIATED METRICS

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On manifolds with a complex almost-product structure, we study some special connections related to the parallelism and integrability of the distributions and to a complex symmetric bilinear form (pseudo-metric) compatible with the structure, and establish the notion of almost-foliated metric which includes as a particular case the metric of a foliated type on a foliated manifold. (For Reinhart spaces see [6].)

1. Adapted connections

Let V be a differentiable manifold of class C^∞ and dimension n, and let $T^C(V) = T(V) \otimes_R C$ denote the complexified space of the tangent space $T(V)$ of the manifold. A complex almost-product structure defined on V gives two C^∞-fields T^1 and T^2 of supplementary subspaces, with respect to the Whitney sum, of $T^C(V)$ ($\dim T^1 = n_1$, $\dim T^2 = n_2, n_1 + n_2 = n$). If $x \in V$, then every vector $X \in T^C_x$ is the sum of two vectors $PX \in T^1_x$ and $QX \in T^2_x$, so that $T^1_x + T^2_x = T^C_x$, $P + Q = I$ (identity), P, Q being the projection tensors associated with T^1 and T^2.

The complex almost-product structure is determined by a vectorial form H such that $H^2 = I$ gives $H = P - Q$ in T^C. It is equivalent to the reduction of the structural group $GL(n, C)$ of the fibration $T^C(V)$. The principal fibration associated with $T^C(V)$ has, as a structural group, the subgroup of the complex linear group $GL(nC)$ of the form

$$(1) \qquad \begin{pmatrix} GL(n_1, C) & 0 \\ 0 & GL(n - n_1, C) \end{pmatrix},$$

The structure determined by the operator $H = P - Q$, such that $H^2 = I$, comprises as particular cases: the almost-complex structure when n is even and $J = iP - iQ$, $\bar{P} = iP$, $\bar{Q} = iQ$ are conjugate operators; and the real almost-product structure when P, Q are real.

We represent by $A(V)$ the fibration of the complex references of T^C with $GL(n, C)$ as the structural group, and by $A'(V)$ the subfibration of the linear references adapted to the complex almost-product structure with (1) as the structural group.

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