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SPHERICAL SPACE FORMS WITH NORMAL CONTACT METRIC 3-STRUCTURE

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Introduction

The theory of contact structure was initiated by S. S. Chern [2] in 1953 in studying pseudo-groups and was developed further by W. M. Boothby & H. C. Wang [1]. G. Reeb [7] also gave an important contribution a little earlier than them to the study of dynamical systems. The generalization to almost contact structure was first studied by J. W. Gray [3] in 1959. The present author [8], [10], [11] with Y. Hatakeyama introduced a new way to study these structures in 1960 by initiating the notions of (almost) contact structure, (almost) contact metric structure, torsion tensor and normality of the structure. Since then many papers on (almost) contact (metric) structures and related topics have been published by many authors.

Recently, Y. Y. Kuo [6] studied Riemannian manifolds with a (almost) contact 3-structure and gave some fundamental properties. Then, S. Tachibana and W. N. Yu [12], S. Tanno [13] and T. Kashiwada [4] studied Riemannian manifolds with a normal contact 3-structure. The purpose of this paper is to study spherical space forms which admit a normal contact metric 3-structure. For the notations on contact structures we refer to the paper [9].

1. Quaternion structure in E^{4d}

1.1. First, let us consider the 4-dimensional case. Let

$$x = x_0 + x_1 i + x_2 j + x_3 k \tag{1.1}$$

be an element of the quaternion algebra Q where x_0, x_1, x_2, x_3 belong to the field of real numbers. We identify x with the vector of components (x_0, x_1, x_2, x_3) of a Euclidean vector space E^4 with respect to an orthonormal basis. Now consider three linear mappings I, J, K of E^4 onto itself defined by

$$Ix = -xi\,, \quad Jx = -xj\,, \quad Kx = -xk\,. \tag{1.2}$$

Then they are complex structures in E^4 and satisfy the relations

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