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HOLOMORPHIC MAPPINGS OF POLYDISCS INTO COMPACT COMPLEX MANIFOLDS

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In this paper we prove an inequality in the manner of the Nevanlinna theory expressing certain properties of holomorphic mappings of n-dimensional polydiscs into compact complex manifolds of the same dimension and discuss some of its applications.

1. Let W be a compact complex manifold of dimension n. For a point w in W, we denote a local coordinate of w by $(w^1, w^2, \cdots, w^n)$. Take a complex line bundle L over W. By a theorem of de Rham, the Chern class $c(L)$ of L can be regarded as a d-cohomology *class* of d-closed 2-forms on W. We say that a real (1, 1)-form

$$\gamma = i \sum_{\alpha,\beta=1}^{n} g_{\alpha\beta}(w) dw^\alpha \wedge d\bar{w}^\beta , \qquad i = \sqrt{-1} ,$$

on W is *positive semidefinite* (*or positive definite*) if the Hermitian matrix $(g_{\alpha\beta}(w))_{\alpha,\beta=1,\cdots,n}$ is positive semidefinite (or positive definite) at every point $w \in W$. Denote the canonical bundle of W by K. In this section we assume the existence of a complex line bundle L over W together with a positive integer m satisfying the following condition: *The Chern class $c(L)$ contains a positive semidefinite d-closed real* (1, 1)*-form and*

$$\dim H^0(W, \mathcal{O}(K^m \otimes L^{-1})) > 0 , \tag{1}$$

where $\mathcal{O}(K^m \otimes L^{-1})$ denotes the sheaf over W of germs of holomorphic sections of $K^m \otimes L^{-1}$.

Cover W by a *finite* number of small neighborhoods U_j, $j = 1, 2, \cdots$, and fix a local coordinate: $w \to (w_j^1, \cdots, w_j^n)$ on each U_j. Take a 1-cocycle $\{l_{jk}\}$ determining the line bundle L composed of nonvanishing holomorphic functions $l_{jk} = l_{jk}(w)$ defined, respectively, on $U_j \cap U_k$. We then find a 0-cochain $\{a_j\}$ composed of C^∞-differentiable functions $a_j = a_j(w) > 0$ defined, respectively, on U_j satisfying

$$a_j(w)^m = |l_{jk}(w)|^2 a_k(w)^m , \qquad \text{on } U_j \cap U_k ,$$

such that

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