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RIGIDITY OF HYPERSURFACES OF CONSTANT SCALAR CURVATURE

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In [7] S. Kobayashi proved that the only compact homogeneous hypersurfaces of a Euclidean space are the spheres. This result was extended by T. Nagano and T. Takahashi [9] who proved that if a homogeneous Riemannian manifold has an isometric immersion in a Euclidean space of one dimension greater such that the rank of the second fundamental form is distinct from two at some point, then it is isometric to the Riemannian product of a sphere by a Euclidean space. The original purpose of this paper was to show that this fact remains true without the restriction on the second fundamental form.

In both [7] and [9], the concept of rigidity has an important role. In fact, if M^n is assumed to be rigid (see preliminaries), the theorem is an immediate consequence of results of E. Cartan [3] and K. Nomizu and B. Smyth [10].

For a homogeneous hypersurface of a Euclidean space, having non-zero constant scalar curvature, there are only two possibilities a priori; it is either rigid or contains no rigid open submanifold (see Corollary 1–8).

The main result of this paper (Theorem 3–1) is that a hypersurface of a Euclidean space, having non-zero constant scalar curvature and containing no open rigid submanifold, is isometric to the product of a two-dimensional sphere and a Euclidean space. This result with the remarks made above gives a proof of Nagano and Takahashi's theorem in the most general case.

The proofs contained in this paper rely heavily on methods developed by E. Cartan [2] and S. Dolbeaut Lemoine [5].

Finally using very similar arguments, the following is proved.

If M^n is a hypersurface of a space form $\tilde{M}^{n+1}(K), n \geq 4$, having constant scalar curvature and an isometric immersion with type number greater than one at all points, then M^n is rigid.

1. All manifolds and maps considered in this work will be assumed of class C^∞. Let M^n be an n-dimensional Riemannian manifold, and denote its tangent space at a point p by T_pM^n and the scalar product given by the Riemannian

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