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MINIMAL SUBMANIFOLDS OF LOW COHOMOGENEITY

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Introduction

Let M be a Riemannian manifold and $I(M)$ its full isometry group. It was shown in [MST] that $I(M)$ is naturally a Lie group which acts differentiably on M. A Lie subgroup G of $I(M)$ is called an *isometry group of* M, and the co-dimension of the maximal dimensional orbits is defined to be the *cohomogeneity of* G. The cohomogeneity of $I(M)$ is called the *cohomogeneity of* M.

Many important Riemannian manifolds such as symmetric spaces, Stiefel manifolds and flag manifolds are of cohomogeneity zero, i.e., homogeneous. The local properties of any such space G/H are completely determined by those at one point and can actually be computed in terms of the infinitesimal structure of the pair $H \subseteq G$. However, many aspects of the global geometry of these spaces (in particular, many questions concerning compact totally geodesic or, more generally, compact minimal submanifolds) are poorly understood. The point of view here is that of using the inherent symmetries of these spaces as a tool in dealing with global questions.

From the point of view of transformation groups the full isometry group G on a Riemannian homogeneous space is too simple, orbit-wise, to be of interest. However, there are many actions induced from the transitive action of G (such as the natural actions on the orthonormal k-frame bundles or the restricted actions of the many subgroups of G) which shed light on the geometric structure of the space. In particular the action of each subgroup gives an interesting decomposition of the space into geodesically parallel orbits. To make proper use of the full isometry group one should study these decompositions, particularly for actions of low cohomogeneity.

The existence and global behavior of compact minimal submanifolds of a homogeneous Riemannian manifold is, in full generality, a difficult area of study. The nonlinearity of the problem makes even the construction of explicit examples reasonably difficult and, at the same time, makes such examples indispensable guidelines for research. Thus, it is natural to try to reduce the complexities of the situation by means of some isometry group G' of low cohomogeneity and, in particular, to look for the existence and general behavior of G'-invariant, minimal submanifolds. That is the purpose here.

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