J. DIFFERENTIAL GEOMETRY
4 (1970) 349-357

RIGIDITY THEOREMS IN RANK-1 SYMMETRIC SPACES

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0. Introduction

Recently, J. Simons proved a number of theorems which in various ways distinguished the geodesic subspheres of Euclidean spheres from all other minimal submanifolds of the sphere. One purpose here is to prove analogous theorems for the totally geodesic submanifolds of CP^k and QP^k-type in the complex and quaternionic projective spaces CP^n and QP^n. In particular, we prove an extrinsic pinching theorem for compact $2k$ (resp. $4k$) dimensional minimal submanifolds of CP^n (resp. QP^n) which leads to an intrinsic rigidity result for the standard imbedding of CP^k (resp. QP^k) within the class of minimal immersions.

We then turn attention to the (real) codimension-one case where, as follows from this work, there are no compact, totally geodesic submanifolds. There are however certain distinguished minimal hypersurfaces $M^C_{p,q}$, $M^Q_{p,q}$ of CP^n and QP^n, for $p, q \geq 0$ and $p + q = n - 1$, which naturally generalize the equatorial hypersurfaces of spheres. We show that there exist positive constants c_n and c'_n such that if M is any compact minimal hypersurface of CP^n (QP^n) over which either the length $\|B\|$ of the second fundamental form satisfies $\|B\| \leq c_n$ or, equivalently, the scalar curvature K satisfies $K \geq c'_n$, then equality holds identically and $M \cong M^C_{p,q}(M^Q_{p,q})$ for some p, q. Moreover, any (not necessarily compact) minimal hypersurface whose scalar curvature is identically equal to c'_n must be an open subset of $M^C_{p,q}$ ($M^Q_{p,q}$).

The method of proof, roughly speaking, is to use the standard fibrations to push known theorems on the sphere down to the spaces CP^n and QP^n.

The author would like to note that results similar to those of Theorem 2 have been found independently by Wu-Hsiung Huang, and also wishes to express gratitude to Wu-Yi Hsiang for many valuable conversations related to this work.

1. Riemannian fibre bundles

Let M' and M be Riemannian manifolds of dimensions m and $m + p$ respectively, and assume that there exists a fibration $\pi\colon M \to M'$ where:

a) The fibres are totally geodesic in M.

Received September 11, 1969.