

TOPOLOGY OF ALMOST CONTACT MANIFOLDS

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Introduction

In his Colloquium Lectures on G -structures [2], S. S. Chern asked for the conditions, both local and global, on a C^∞ manifold in order that a linear differential form η exist such that

$$\eta \wedge (d\eta)^p \neq 0$$

for a given value of p . The form η defines a differential system and it is important to study the local and global properties of its integral manifolds. To this end, the notion of a quasi-Sasakian structure on an almost contact metric manifold was introduced by one of the authors [1] and its main properties developed. In the present paper their topological properties are considered and it is shown that both compact Sasakian and cosymplectic manifolds have global properties similar to compact Kaehler manifolds. Examples are the unit hypersphere S^{2n+1} in Euclidean space, and in fact, the circle bundles over any compact Hodge variety. In the latter class, examples are provided by $M \times S^1$ where M is any compact Kaehler manifold. As one might expect, therefore, not only locally, but topologically as well, the compact cosymplectic spaces are the proper odd dimensional analogues of the compact Kaehler manifolds. A complete, but not compact, simply connected cosymplectic manifold is a product with one factor Kaehlerian.

The notation and terminology in this paper will be the same as that employed in [1].

1. Topology of Sasakian manifolds

Define two operators L and Λ , dual to each other, on a quasi-Sasakian manifold by $L = \varepsilon(\Phi)$ and $\Lambda = \iota(\Phi)$ where ε and ι are respectively the exterior and interior product operators. We say that a p -form α ($p \geq 2$) is *effective* if $\Lambda\alpha = 0$. Since $\iota(\Phi) = *\varepsilon(\Phi)*$ where $*$ is the Hodge star isomorphism, $\Lambda = *L*$.

An orthonormal basis of \mathcal{E}^{2n+1} on an almost contact metric manifold M^{2n+1} of the form $\{\xi, X_i, X_{i*} = \phi X_i\}$, $i = 1, \dots, n$, is called a ϕ -basis. It is well known that such a basis always exists. For, let $V = \{X \in M_m \mid g(X, \xi) = 0\}$. Equations (1.1) and (1.2) of [1] show that $\phi|_V$ is an almost complex structure

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