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LEFSCHETZ PENCILS ON SYMPLECTIC MANIFOLDS

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This paper is a sequel to [3], in which techniques from complex geometry were adapted to prove a general existence theorem for symplectic submanifolds of compact symplectic manifolds. These submanifolds were obtained as the zero-sets of suitable sections of complex line bundles. In the present paper we take the ideas further, developing the symplectic analogue of "pencils", generated by a pair of sections of a line bundle. Our main results are a general existence theorem for topological Lefschetz pencils (Theorem 2 below), together with an asymptotic uniqueness statement (Theorem 20). These results, along with recent work of R. Gompf ([4, Theorem 10.2.18]), go some way towards giving a topological characterisation of symplectic manifolds; more generally they, along with various further extensions, give a means of translating many questions in symplectic topology into questions about the "monodromy" of the pencil. We will leave the discussion of these further topics for future papers, and concentrate here on the proofs of the main existence theorems.

The techniques of [3] were developed further by D. Auroux [1], [2]. In particular, Auroux proved the asymptotic uniqueness of the submanifolds constructed in [3], and there are many areas of overlap between Auroux' work and that in this paper. Our approach to the proofs is to throw the main burden of work onto a transversality result (Theorem 12) for holomorphic maps, extending the corresponding result in [3]. With this result to hand, the rest of the proof is a fairly straightforward extension of the argument in [3]. It is probably possible to arrange the

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