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POLAR ACTIONS ON RANK-ONE SYMMETRIC SPACES

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Abstract

We give a complete classification up to orbit equivalence of polar actions of compact connected Lie groups on compact rank-one symmetric spaces. For polar actions on the complex projective spaces, we prove that they are orbit equivalent to the actions induced by isotropy representations of Hermitian symmetric spaces, while in the case of polar actions on the quaternionic projective spaces, we prove that they are orbit equivalent to the actions induced by products of k quaternion-Kähler symmetric spaces, where at least $k-1$ have rank one. For the Cayley projective plane $\mathbb{P}_2(\mathbb{O})$, we prove that the cohomogeneity of any polar action is either one or two, and we come up with a complete list of all compact connected subgroups of F_4 (up to conjugacy) acting polarly on $\mathbb{P}_2(\mathbb{O})$. The classification of polar actions on spheres and real projective spaces follows immediately from Dadok's paper [10].

0. Introduction

An isometric action of a compact Lie group G on a Riemannian manifold M is called *polar* if there exists a properly embedded, connected submanifold which meets every G-orbit orthogonally; any such submanifold is called a *section* and if the induced metric on a section is flat, then the action is called *hyperpolar*.

One should think of a section as a set of canonical forms for the polar action; see [24]. An example that demonstrates this viewpoint is the conjugation of symmetric matrices by elements of the orthogonal group. This action is polar with the diagonal matrices as a section.

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