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ON THE QUANTUM EXPECTED VALUES OF INTEGRABLE METRIC FORMS

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1. Introduction

Let (M^n, g) be a compact, real-analytic, Riemannian manifold, P_0 a first order, self-adjoint, real-analytic, elliptic pseudodifferential operator with principal symbol,

$$H(x,\xi) = \sqrt{g^{ij}(x)\xi_i\xi_j}$$

generating geodesic flow. We will assume that P_0 is quantum integrable; that is, there exist n-1 first order, jointly elliptic, real-analytic, classical pseudodifferential operators P_1, \ldots, P_{n-1} such that, for all $i, j = 0, 1, \ldots, n-1$,

$$[1) \qquad \qquad [P_i, P_j] = 0.$$

Given the Hamilton vector field,

$$\Xi_H = \sum_{j=1}^n \frac{\partial H}{\partial \xi_j} \frac{\partial}{\partial x_j} - \frac{\partial H}{\partial x_j} \frac{\partial}{\partial \xi_j},$$

we denote the associated geodesic flow by $\exp t\Xi_H : C^{\infty}(S^*M) \to C^{\infty}(S^*M)$. Suppose γ is a simple, periodic orbit of $\exp t\Xi_H$ (i.e., a

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