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ON THE CONVERGENCE AND COLLAPSING OF KÄHLER METRICS

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Abstract

In this paper we consider the convergence and collapsing of Kähler manifolds. While the convergence and collapsing of Riemannian manifolds have been discussed by many people and applied to many fields, how to generalize it to Kähler case is not apriorily clear. Our paper is an attempt in this direction. We discussed the corresponding concepts of convergence and collapsing for Kähler manifolds. We proved that when a sequence of Kähler manifolds with the fixed background complex compact manifold is not collapsing, it will converge to a complete Kähler manifold which is biholomorphic to a Zariski open set of the original background complex manifold with some possible "bubbling" on the complement of that Zariski open set. We also discussed the structure of collapsing. Especially we show the resulting Monge-Ampère foliation is holomorphic, produce some holomorphic vector fields with respect to the foliation, and also give some applications of our results. The main methods we are using are estimates from the theories of harmonic maps and partial differential equations, some results from several complex variables, and ideas from Riemannian geometry.

1. Introduction and background

Given a compact complex manifold M with a fixed complex structure, for a given sequence of Kähler metrics $\{g_i\}$ on M, consider $\{(M, g_i)\}$ as a sequence of Kähler manifolds. We want to study how do they converge.

In the most generality, one has the following Gromov-Hausdorff convergence. On the space of compact metric spaces, define

$$d(A,B) = inf \left\{ \epsilon \;\middle|\; \begin{array}{c} A \hookrightarrow X \hookleftarrow B, \text{isometric embedding,} \\ A \subset U(B,\epsilon), B \subset U(A,\epsilon) \end{array} \right\}$$

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