J. DIFFERENTIAL GEOMETRY
54 (2000) 255-302

THE ISOPERIMETRIC PROFILE OF HOMOGENEOUS RIEMANNIAN MANIFOLDS

CH. PITTET

Abstract

We compute, up to a multiplicative constant, the isoperimeric profile of (non-compact) homogeneous Riemannian manifolds by constructing "explicit" exhaustions which give estimates for the distribution of the volume. For those Riemannian manifolds only three very different isoperimetric profiles exist and the isoperimetric profile governs the asymptotic of the heat kernel decay on the diagonal and vice-versa. By discretisation, the isoperimetric profiles of finitely generated discrete subgroups of Lie groups are also computed.

1. Introduction

1.1 The classical isoperimetric profile

Let X be a complete Riemannian manifold and let $0 < t < vol(X)$. The isoperimetric profile of X is

$$I_X(t) = \inf_{vol(\Omega)=t} vol(\partial\Omega),$$

where the infimum is taken over relatively compact domains Ω with regular boundary $\partial\Omega$. Compare [5, pp. 140-143], [16, 4.74]. With the exception of the simply connected constant sectional curvature spaces $\mathbb{R}^n, S^n, H^n$ where the infimum is realised by balls, the exact computation of $I_X(t)$ even for familiar Riemannian manifolds may be problematic. For example, in the case of real projective spaces with the locally spherical metric, the answer is known and proved only in dimension 2 and

Received February 4, 2000.
1991 *Mathematics Subject Classification.* 53C23, 53C30.