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CORRECTION TO "MODULI SPACES OF NONNEGATIVE SECTIONAL CURVATURE AND NON-UNIQUE SOULS" [JOURNAL OF DIFFERENTIAL GEOMETRY 89 (2011), NO. 1, 49–85.]

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The purpose of this note is to report two misstatements in [1] which, luckily, has affected no other result in the literature.

Proposition 2.8 in [1] claims that the map **soul** is a homeomorphism, but only establishes its continuity, where the topology is C^k the domain and C^{k_0} on the co-domain with $k_0 = \max\{0, k-1\}$. The inverse of **soul** is continuous if and only if $k = k_0$, i.e., k is zero or infinity. The other results in [1, 2] are unaffected since they assume $k = \infty$. A correction for $k \geq 2$ can be found in [8, Corollary 2.2].

In [1, Proposition 4.4] we inadvertently omitted an assumption, which was satisfied in all applications of the proposition in [1, 2]. To describe the omission let ξ, η be smooth vector bundles over closed n-manifolds B_ξ, B_η sharing the same total space, and consider the map $f_{\xi,\eta}\colon B_\xi \to B_\eta$ obtained by composing the zero section of ξ with the projection of η. Proposition 4.4 claims that if $f_{\xi,\eta}$ pulls η back to ξ, then $f_{\xi,\eta}$ has trivial normal invariant. This was used in [1, 2] only when ξ has 2-dimensional fibers, and under this assumption the claim is true. Furthermore, in this case the assumption that $f_{\xi,\eta}$ pulls η back to ξ can be dropped, namely, we shall prove below that *if ξ has 2-dimensional fibers, then $f_{\xi,\eta}$ has trivial normal invariant, and $f_{\xi,\eta}$ pulls η to ξ.* Note that either conclusion implies that $f_{\xi,\eta}$ is tangential, i.e., it preserves the stable tangent bundle.

A counterexample to the original statement of Proposition 4.4 is given by any pair of closed smooth simply-connected manifolds B_1, B_2 of dimension ≥ 5 that are tangentially homotopy equivalent and non-homeomorphic. In this case $B_1 \times \mathbb{R}^k$, $B_2 \times \mathbb{R}^k$ are diffeomorphic for every sufficiently large k, and any homotopy equivalence of B_1 and B_2 clearly preserves the trivial normal bundles. If the homotopy equivalence had trivial normal invariant, then B_1, B_2 would be homeomorphic, see [3, Corollary II.3.8]. In fact, there even exist B_1, B_2 as above with metrics of nonnegative curvature which was the setting of [1]; such examples can be found in [5, p. 114], [4, Proposition 5.6], and [6].

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