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A CONSTRUCTION OF INFINITELY MANY SOLUTIONS TO THE STROMINGER SYSTEM

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1. Introduction

The Strominger system [**23**, **37**] is a system of partial differential equations characterizing the compactification of heterotic superstrings with torsion. Mathematically speaking, we may think of the Strominger system as a generalization of Ricci-flat metrics on non-Kähler Calabi-Yau 3-folds, which is simultaneously coupled with the Hermitian Yang-Mills equation on a gauge bundle.

Let X be a complex 3-fold, preferably compact, with holomorphically trivial canonical bundle. We fix a nowhere vanishing holomorphic (3,0)-form Ω on X. Let ω be a Hermitian metric on X, and denote by $\|\Omega\|_\omega$ the norm of Ω with respect to the metric ω. In addition, let (E,h) be a holomorphic vector bundle over X equipped with a Hermitian metric. We denote by R and F the endomorphism-valued curvature 2-forms of the holomorphic tangent bundle $T^{1,0}X$ and E respectively. Finally, let $\alpha' \in \mathbb{R}$ be a constant. We can write down the Strominger system as follows [**24**]:

$$F \wedge \omega^2 = 0, \quad F^{0,2} = F^{2,0} = 0, \tag{1}$$

$$i\partial\bar{\partial}\omega = \frac{\alpha'}{4}(\mathrm{Tr}(R\wedge R) - \mathrm{Tr}(F\wedge F)), \tag{2}$$

$$\mathrm{d}(\|\Omega\|_\omega \cdot \omega^2) = 0. \tag{3}$$

In the literature, Equations (1), (2) and (3) are known as the Hermitian Yang-Mills equation, the anomaly cancellation equation and the conformally balanced equation respectively. In this paper, we will only use the Chern connection to compute the curvatures R and F and work with positive α'.

It is not hard to see that when ω is a Kähler metric, by Yau's theorem [**41**] the full Strominger system can be solved if we take ω to be Ricci-flat and simultaneously embed the spin connection into the Yang-Mills connection, meaning to set $R = F$. Such a solution corresponds to the torsion-free compactification of superstrings [**4**]. However, the Strominger system allows for more general backgrounds than Kähler Calabi-Yau manifolds. In fact, the interest of the Strominger system is

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