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3D INCOMPRESSIBLE FLUIDS: COMBINATORIAL MODELS, EIGENSPACE MODELS, AND A CONJECTURE ABOUT WELL-POSEDNESS OF THE 3D ZERO VISCOSITY LIMIT

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Dedicated to Fritz Hirzebruch,
who exemplified diligence and beauty in mathematics

Part 1. Combinatorial Models for Computation

1. Introduction

We make combinatorial models of spatial calculus with special regard for nonlinear structures. We apply this to incompressible fluid motion in the zero viscosity limit in the 3-dimensional space made periodic. We take advantage of a special duality property of the cubical partitions of 3-dimensional space. The nonlinear structure comes from the evolution PDE of fluids

$$\dot{Y} = [X, Y],$$

where X is a divergence free vector field (i.e., incompressible), $Y = \mathrm{curl} X$, and $[\ ,\ ]$ is the Lie bracket which is our nonlinear structure. This equation states that *the vorticity of the fluid motion is transported by the motion of the fluid*.

We use the powerful tools of algebraic topology to somewhat open up the structure of the nonlinear term. In order to use these tools it is necessary to embed vector fields on a smooth manifold into the chain complex of multivector fields with natural monomial grading and boundary operator of degree -1. This structure appears by regarding multivector fields as linear functionals on smooth differential forms. To do this it is enough to choose any smooth probability measure that charges every open set. The point is that the Lie bracket is now intertwined with this chain complex structure as explained below.

2. The homotopy category of chain complexes

By a *chain complex* C we mean a family $C = \{C_i\}_{i \in \mathbb{Z}}$ of real vector spaces together with linear maps $\partial = \partial_i : C_i \to C_{i-1}$ such that the

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