J. DIFFERENTIAL GEOMETRY
87 (2011) 239-307

LOCALLY STRONGLY CONVEX AFFINE HYPERSURFACES WITH PARALLEL CUBIC FORM

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Abstract

We give a complete classification of locally strongly convex affine hypersurfaces of $\mathbb{R}^{n+1}$ with parallel cubic form with respect to the Levi-Civita connection of the affine Berwald-Blaschke metric. It turns out that all such affine hypersurfaces are quadrics or can be obtained by applying repeatedly the Calabi product construction of hyperbolic affine hyperspheres, using as building blocks either the hyperboloid, or the standard immersion of one of the symmetric spaces $\mathbf{SL}(m,\mathbb{R})/\mathbf{SO}(m)$, $\mathbf{SL}(m,\mathbb{C})/\mathbf{SU}(m)$, $\mathbf{SU}^*(2m)/\mathbf{Sp}(m)$, or $\mathbf{E}_{6(-26)}/\mathbf{F}_4$.

1. Introduction

In this paper, we study affine hypersurfaces of $\mathbb{R}^{n+1}$. The study of affine differential geometry originates with the work of Blaschke and his coworkers at the beginning of the twentieth century [**Bl**]. A more modern structural approach to this field was given by Nomizu at the 1984 conference Differential Geometry Meeting in Münster [**N**].

In the case that the hypersurface is nondegenerate, it is well known how to induce an affine connection ∇ and a symmetric bilinear form h, called the affine metric, on M. This is done by constructing a canonical transversal vector field to the immersion, called the affine normal. The classical Pick-Berwald theorem states that the induced affine connection coincides with the Levi-Civita connection of the affine metric if and only if the hypersurface is a quadric. For that reason, the difference tensor

$$K(X,Y) = \nabla_X Y - \hat{\nabla}_X Y,$$

where $\hat{\nabla}$ is the Levi-Civita connection of the affine metric, plays a fundamental role in affine differential geometry.

Here in this paper, we will always assume that the hypersurface is locally strongly convex, i.e., the affine metric is definite. In this case, if necessary by changing the sign of the affine normal, we may always assume that the affine metric is positive definite. In particular, we will

Received 11/5/2009.