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IMMERSED LAGRANGIAN FLOER THEORY

Manabu Akaho & Dominic Joyce

Abstract

Let (M,ω) be a compact symplectic manifold, and L a compact embedded Lagrangian submanifold in M. Fukaya, Oh, Ohta and Ono [**8**] construct *Lagrangian Floer cohomology*, yielding groups $HF^*(L,b;\Lambda_{\rm nov})$ for one or $HF^*\big((L_1,b_1),(L_2,b_2);\Lambda_{\rm nov}\big)$ for two Lagrangians, where b,b_1,b_2 are choices of *bounding cochains*, and exist if and only if L,L_1,L_2 have *unobstructed Floer cohomology.* These are independent of choices up to isomorphism, and have important invariance properties under Hamiltonian equivalence. Floer cohomology groups are the morphism groups in the derived Fukaya category of (M,ω), and so are an essential part of the Homological Mirror Symmetry Conjecture of Kontsevich.

The goal of this paper is to extend [**8**] to *immersed* Lagrangians $\iota:L\rightarrow M$, with transverse self-intersections. In the embedded case, Floer cohomology $HF^*(L,b;\Lambda_{\rm nov})$ is a modified, 'quantized' version of singular homology $H_{n-*}(L;\Lambda_{\rm nov})$ over the Novikov ring $\Lambda_{\rm nov}$. In our immersed case, $HF^*(L,b;\Lambda_{\rm nov})$ turns out to be a quantized version of $H_{n-*}(L;\Lambda_{\rm nov})\oplus\bigoplus_{(p_-,p_+)\in R}\Lambda_{\rm nov}\cdot(p_-,p_+)$, where $R=\big\{(p_-,p_+):p_-,p_+\in L,\ p_-\neq p_+,\ \iota(p_-)=\iota(p_+)\big\}$ is a set of two extra generators for each self-intersection point of L, and (p_-,p_+) has degree $\eta_{(p_-,p_+)}\in\mathbb{Z}$, an index depending on how L intersects itself at $\iota(p_-)=\iota(p_+)$.

The theory becomes simpler and more powerful for *graded* Lagrangians in Calabi–Yau manifolds, when we can work over a smaller Novikov ring $\Lambda_{\rm CY}$. The proofs involve associating a gapped filtered A_∞ algebra over $\Lambda_{\rm nov}^0$ or $\Lambda_{\rm CY}^0$ to $\iota:L\rightarrow M$, which is independent of nearly all choices up to canonical homotopy equivalence, and is built using a series of finite approximations called $A_{N,0}$ algebras for $N=0,1,2,\ldots$.

1. Introduction

Let (M,ω) be a compact symplectic manifold, and L a compact embedded Lagrangian submanifold in M. Fukaya, Oh, Ohta and Ono

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