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EXPLICIT BIRATIONAL GEOMETRY OF 3-FOLDS OF GENERAL TYPE, II

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Abstract

Let V be a complex nonsingular projective 3-fold of general type. We shall give a detailed classification up to baskets of singularities on a minimal model of V. We show that the m-canonical map of V is birational for all $m \geq 73$ and that the canonical volume $\mathrm{Vol}(V) \geq \frac{1}{2660}$. When $\chi(\mathcal{O}_V) \leq 1$, our result is $\mathrm{Vol}(V) \geq \frac{1}{420}$, which is optimal. Other effective results are also included in the paper.

1. Introduction

Let Y be a nonsingular projective variety of dimension n. It is said to be of general type if the pluricanonical map φ_m corresponding to the linear system $|mK_Y|$ is birational into a projective space for $m \gg 0$. Thus it is natural and important to find a constant $c(n)$, depending only on dimension, so that φ_m is birational onto its image for all $m \geq c(n)$ and for all Y with $\dim Y = n$.

It was classically known that, when $\dim Y = 1$, $|mK_Y|$ gives an embedding of Y into a projective space for $m \geq 3$. When $\dim Y = 2$, Kodaira-Bombieri's theorem [**2**] implies that $|mK_Y|$ gives a birational map onto the image for $m \geq 5$. A recent result of Hacon and M$^{\mathrm{c}}$Kernan [**10**], Takayama [**23**], and Tsuji [**25**] shows the existence of $c(n)$, which is however non-explicit.

This is the continuation of our previous paper [**4**]. The aim of this paper is to prove a practical constant $c(3)$, which is not too far from being sharp. Other effective results are included in this paper as well.

Recall that we have proved the following result in [**4**].

Theorem 1. ([**4**, Theorem 1.1]) *Let V be a nonsingular projective 3-fold of general type. Then:*

(1) $P_{12} > 0$;
(2) $P_{m_0} \geq 2$ *for some positive integer* $m_0 \leq 24$.

Our main theorems of this paper are as follows.

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