

HYPERPOLAR HOMOGENEOUS FOLIATIONS ON SYMMETRIC SPACES OF NONCOMPACT TYPE

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Abstract

A foliation \mathcal{F} on a Riemannian manifold M is hyperpolar if it admits a flat section, that is, a connected closed flat submanifold of M that intersects each leaf of \mathcal{F} orthogonally. In this article we classify the hyperpolar homogeneous foliations on every Riemannian symmetric space M of noncompact type.

These foliations are constructed as follows. Let Φ be an orthogonal subset of a set of simple roots associated with the symmetric space M . Then Φ determines a horospherical decomposition $M = F_{\Phi}^s \times \mathbb{E}^{\text{rank } M - |\Phi|} \times N_{\Phi}$, where F_{Φ}^s is the Riemannian product of $|\Phi|$ symmetric spaces of rank one. Every hyperpolar homogeneous foliation on M is isometrically congruent to the product of the following objects: a particular homogeneous codimension one foliation on each symmetric space of rank one in F_{Φ}^s , a foliation by parallel affine subspaces on the Euclidean space $\mathbb{E}^{\text{rank } M - |\Phi|}$, and the horocycle subgroup N_{Φ} of the parabolic subgroup of the isometry group of M determined by Φ .

1. Introduction

Let M be a connected complete Riemannian manifold and H a connected closed subgroup of the isometry group $I(M)$ of M . Then each orbit $H \cdot p = \{h(p) : h \in H\}$, $p \in M$, is a connected closed submanifold of M . A connected complete submanifold \mathcal{S} of M that meets each orbit of the H -action and intersects the orbit $H \cdot p$ perpendicularly at each point $p \in \mathcal{S}$ is called a section of the action. A section \mathcal{S} is always a totally geodesic submanifold of M (see e.g. [11]). In general, actions do not admit a section. The action of H on M is called polar if it has a section, and it is called hyperpolar if it has a flat section. For motivation and classification of polar and hyperpolar actions on Euclidean spaces

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