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AN EXTENSION OF THE STABILITY THEOREM OF THE MINKOWSKI SPACE IN GENERAL RELATIVITY

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Abstract

In this paper, we sketch the proof of the extension of the stability theorem of the Minkowski space in General Relativity done explicitly in [**6**] and [**7**]. We discuss solutions of the Einstein vacuum (EV) equations (obtained in the author's Ph.D. thesis ([**6**]) in 2007). We solve the Cauchy problem for more general, asymptotically flat initial data than in the pioneering work of D. Christodoulou and S. Klainerman [**21**] or than in any other work. Moreover, we describe precisely the asymptotic behavior. Our relaxed assumptions on the initial data yield a spacetime curvature which is not bounded in $L^\infty(M)$. As a major result, we encounter in our work borderline cases, which we discuss in this paper as well. The fact that certain of our estimates are borderline in view of decay indicates that the conditions in our main theorem are sharp in so far as the assumptions on the decay at infinity on the initial data are concerned. Thus, the borderline cases are a consequence of our relaxed assumptions on the data, [**6**, **7**]. They are not present in the other works, as all of them place stronger assumptions on their data. We work with an invariant formulation of the EV equations. Our main proof is based on a bootstrap argument. To close the argument, we have to show that the spacetime curvature and the corresponding geometrical quantities have the required decay. In order to do so, the Einstein equations are decomposed with respect to specific foliations of the spacetime. This result generalizes the work of D. Christodoulou and S. Klainerman [**21**].

1. Introduction and Main Results

The laws of General Relativity (GR) are the Einstein equations linking the curvature of the spacetime to its matter content.

$$G_{\mu\nu} \;:=\; R_{\mu\nu} \;-\; \frac{1}{2}\, g_{\mu\nu}\, R \;=\; 2\, T_{\mu\nu}, \tag{1}$$

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