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MEAN CURVATURE FLOW OF PINCHED SUBMANIFOLDS TO SPHERES

Ben Andrews & Charles Baker

Abstract

We consider compact submanifolds of dimension $n \geq 2$ in $\mathbb{R}^{n+k}$, with nonzero mean curvature vector everywhere, and such that the full norm of the second fundamental form is bounded by a fixed multiple (depending on n) of the length of the mean curvature vector at every point. We prove that the mean curvature flow deforms such a submanifold to a point in finite time, and that the solution is asymptotic to a shrinking sphere in some $(n+1)$-dimensional affine subspace of $\mathbb{R}^{n+k}$.

1. Introduction

The evolution of hypersurfaces by their mean curvature has been studied by many authors since the appearance of Gerhard Huisken's seminal paper [**Hu1**]. More recently, mean curvature flow of higher codimension submanifolds has also received attention. In this paper we prove a result analogous to that of [**Hu1**] for submanifolds of any codimension.

Let $F_0 : \Sigma^n \to \mathbb{R}^{n+k}$ be a smooth immersion of a compact manifold Σ. The mean curvature flow with initial condition F_0 is a smooth family of immersions $F : \Sigma \times [0, T) \to \mathbb{R}^{n+k}$ satisfying

$$(1) \qquad \begin{cases} \frac{\partial}{\partial t} F(p,t) = H(p,t), \quad p \in \Sigma, \ t \geq 0, \\ F(\cdot, 0) = F_0, \end{cases}$$

where $H(p,t)$ is the mean curvature vector of the submanifold $\Sigma_t = F(\Sigma, t)$ at p. We use the abbreviation "MCF" for the system (1), and denote the second fundamental form by h. See section 2 for further details of our notation and conventions.

High codimension MCF is the steepest descent flow for the area functional, and so arises naturally in several contexts. For example, singular sets in harmonic map heat flow move by generalized mean curvature flow

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