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THE DYNAMICS THEOREM FOR CMC SURFACES IN $\boldsymbol{R}^3$

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Abstract

In this paper, we study the space of translational limits $\mathcal{T}(M)$ of a surface M properly embedded in $\mathbb{R}^3$ with nonzero constant mean curvature and bounded second fundamental form. There is a natural map $\mathcal{T}$ which assigns to any surface $\Sigma \in \mathcal{T}(M)$ the set $\mathcal{T}(\Sigma) \subset \mathcal{T}(M)$. Among various dynamics type results we prove that surfaces in minimal $\mathcal{T}$-invariant sets of $\mathcal{T}(M)$ are chord-arc. We also show that if M has an infinite number of ends, then there exists a nonempty minimal $\mathcal{T}$-invariant set in $\mathcal{T}(M)$ consisting entirely of surfaces with planes of Alexandrov symmetry. Finally, when M has a plane of Alexandrov symmetry, we prove the following characterization theorem: M has finite topology if and only if M has a finite number of ends greater than one.

1. Introduction

A general problem in classical surface theory is to describe the asymptotic geometric structure of a connected, noncompact, properly embedded, nonzero constant mean curvature (CMC) surface M in $\mathbb{R}^3$. In this paper, we will show that when M has bounded second fundamental form, for any divergent sequence of points $p_n \in M$, a subsequence of the translated surfaces $M - p_n$ converges to a properly immersed surface of the same constant mean curvature which bounds a smooth open subdomain on its mean convex side. The collection $\mathbb{T}(M)$ of all these limit surfaces sheds light on the geometry of M at infinity.

We will focus our attention on the subset $\mathcal{T}(M) \subset \mathbb{T}(M)$ consisting of the connected components of surfaces in $\mathbb{T}(M)$ which pass through the origin in $\mathbb{R}^3$. Given a surface $\Sigma \in \mathcal{T}(M)$, we will prove that $\mathcal{T}(\Sigma)$ is always a subset of $\mathcal{T}(M)$. In particular, we can consider $\mathcal{T}$ to represent a function:

$$\mathcal{T} \colon \mathcal{T}(M) \to \mathcal{P}(\mathcal{T}(M)),$$

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