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# ON A CHARACTERIZATION OF THE COMPLEX HYPERBOLIC SPACE

Ovidiu Munteanu


## Abstract

Consider a compact Kähler manifold $M^m$ with Ricci curvature lower bound $Ric_M \geq -2\,(m+1)$. Assume that its universal cover $\widetilde{M}$ has maximal bottom of spectrum $\lambda_1\left(\widetilde{M}\right) = m^2$. Then we prove that $\widetilde{M}$ is isometric to the complex hyperbolic space $\mathbb{CH}^m$.


## 1. Introduction

Complete Riemannian manifolds with negative Ricci curvature lower bound have been investigated by many authors. An important approach is to see how the spectrum of the Laplacian interacts with the geometry of the manifold. A classical result that we recall here is S.Y. Cheng's comparison theorem [**C**], which states that the hyperbolic space $\mathbb{H}^n$ has the greatest bottom of spectrum among all complete Riemannian manifolds with Ricci curvature at least the Ricci curvature of $\mathbb{H}^n$.

Therefore, if the Ricci curvature of a complete noncompact Riemannian manifold $N^n$ of dimension $n$ is bounded below by $Ric_N \geq -\,(n-1)$, then the bottom of the spectrum of the Laplacian has an upper bound $\lambda_1\,(N) \leq \frac{(n-1)^2}{4}$. This result is sharp, but we should point out that there are in fact many manifolds with maximal $\lambda_1$, and more examples can be found by considering hyperbolic manifolds $N = \mathbb{H}^n/\Gamma$ obtained by the quotient of $\mathbb{H}^n$ by a Kleinian group $\Gamma$ (see [**S**]).

While in general we cannot determine the class of manifolds with $\lambda_1$ achieving its maximal value, recently there has been important progress in some directions.

P. Li and J. Wang have studied the structure at infinity of a complete noncompact Riemannian manifold that has $Ric_N \geq -\,(n-1)$ and maximal bottom of spectrum $\lambda_1\,(N) = \frac{(n-1)^2}{4}$. They proved that either the manifold is connected at infinity (i.e., it has one end) or it has two ends. In the case where it has two ends, it must split as a warped product of a compact manifold with the real line [**L-W2**]. Their result has

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