J. DIFFERENTIAL GEOMETRY
84 (2010) 127-161

SELF-SIMILAR SOLUTIONS AND TRANSLATING SOLITONS FOR LAGRANGIAN MEAN CURVATURE FLOW

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Abstract

We construct many self-similar and translating solitons for Lagrangian mean curvature flow, including self-expanders and translating solitons with arbitrarily small oscillation on the Lagrangian angle. Our translating solitons play the same role as cigar solitons in Ricci flow, and are important in studying the regularity of Lagrangian mean curvature flow.

Given two transverse Lagrangian planes $\mathbb{R}^n$ in $\mathbb{C}^n$ with sum of characteristic angles less than π, we show there exists a Lagrangian self-expander asymptotic to this pair of planes. The Maslov class of these self-expanders is zero. Thus they can serve as local models for surgeries on Lagrangian mean curvature flow. Families of self-shrinkers and self-expanders with different topologies are also constructed. This paper generalizes the work of Anciaux [**1**], Joyce [**12**], Lawlor [**15**], and Lee and Wang [**18**, **19**].

1. Introduction

Special Lagrangian submanifolds in Calabi–Yau n-folds have received much attention in recent years, as they are key ingredients in the Strominger–Yau–Zaslow Conjecture [**25**], which explains Mirror Symmetry of Calabi–Yau 3-folds. Thomas and Yau [**26**] defined a notion of stability for graded Lagrangians L in a Calabi–Yau n-fold M, and conjectured that if L is stable then the *Lagrangian mean curvature flow* of L exists for all time and converges to a special Lagrangian submanifold L_∞ in M, which should be the unique special Lagrangian in the Hamiltonian equivalence class of L.

Rewriting this in terms of the derived Fukaya category $D^b\mathrm{Fuk}(M)$ of M, as in Kontsevich's Homological Mirror Symmetry program [**14**], and using Bridgeland's notion of stability condition on triangulated categories [**4**], one can state an improved (but still over-simplified) version of the Thomas–Yau conjecture as follows: for any Calabi–Yau n-fold M, there should exist a Bridgeland stability condition $(Z,\mathcal{P})$ on $D^b\mathrm{Fuk}(M)$ depending on the holomorphic $(n,0)$-form Ω on M, such that a graded Lagrangian L in M is $(Z,\mathcal{P})$-stable, regarded as an object in $D^b\mathrm{Fuk}(M)$,