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A HOPF THEOREM FOR AMBIENT SPACES OF DIMENSIONS HIGHER THAN THREE

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Abstract

We consider surfaces M^2 immersed in $E_c^n \times \mathbb{R}$, where E_c^n is a simply connected n-dimensional complete Riemannian manifold with constant sectional curvature $c \neq 0$, and assume that the mean curvature vector of the immersion is parallel in the normal bundle. We consider further a Hopf-type complex quadratic form Q on M^2, where the complex structure of M^2 is compatible with the induced metric. It is not hard to check that Q is holomorphic (see [**3**], p.289). We will use this fact to give a reasonable description of immersed surfaces in $E_c^n \times \mathbb{R}$ that have parallel mean curvature vector.

1. Introduction

A beautiful result on surfaces M^2 immersed in a 3-dimensional euclidean space $\mathbb{R}^3$ was obtained by H. Hopf in 1951 [**6**] and states that if M^2 is homeomorphic to a sphere and has constant mean curvature H, then M^2 is totally umbilic, hence isometric to a round sphere. The basic idea of Hopf's proof is to introduce a complex quadratic form $\tilde{\alpha}$ in M^2 (in the complex structure of M^2 determined by its induced metric) and prove that $\tilde{\alpha}$ is holomorphic if $H =$ constant. Hopf's theorem was extended by Chern [**4**] to surfaces immersed in a 3-dimensional Riemannian manifold M_c^3 (we use superscripts to denote dimensions) with constant sectional curvature c and, recently, for surfaces in simply-connected, homogeneous 3-dimensional Riemannian manifolds with a 4-dimensional group of isometries (Abresch and Rosenberg [**1**], [**2**]).

It is then natural to look for higher dimensional ambient spaces in which a Hopf-type theorem holds.

In this paper, we study the case where $x\colon M^2 \to E_c^n \times \mathbb{R}$ is a surface immersed in the product Riemannian manifold of a simply-connected n-dimensional Riemannian manifold E_c^n of constant sectional curvature $c \neq 0$ with the euclidean line $\mathbb{R}$. We assume that the mean curvature

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