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ADDENDUM TO "PRINCIPAL BUNDLES ON PROJECTIVE VARIETIES AND THE DONALDSON-UHLENBECK COMPACTIFICATION"

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Abstract

In this addendum we complete the proof of Theorem 7.10 in [**1**].

In the paper [**1**], the proof of Theorem 7.10 is incomplete. This theorem proves the asymptotic non-emptiness of the moduli space of stable G–bundles on smooth projective surfaces, with structure group G which is simple and simply connected. This was pointed to us by Prof. S. Ramanan. In this addendum we complete the proof of this theorem. We retain the notations of [**1**].

The results in [**1**] which need to be modified are Lemma 7.5 and the proof of Proposition 7.6. In the cases which were considered in the proof of [**1**, Proposition 7.6], the case when the monodromy group of a rank 2 stable bundle is the normalizer of a one-dimensional torus in $SL(2)$ was not taken into consideration. When this case is allowed, the loci of bundles in $M_C(SL(2))^s$, where the monodromy groups are proper subgroups of $SL(2)$, is no longer a countable set. In fact, it contains a subvariety whose dimension is $g(C) - 1$, $g(C)$ being the genus of C. We therefore cannot work with the restriction to general curves as was done earlier in [**1**]. Instead, we use a technique due to S. Donaldson (cf. [**2**]) which completes the argument without much difficulty.

Recall from [**1**, Page 393] that we wish to estimate the dimension of subset Z_C of representations of $\pi_1(C)$ in $SU(2)$ which lie entirely in these families of groups up to conjugacy by the diagonal action of $SU(2)$.

Since all the cases except $\mathcal{M}(V) = N(T)$ have been handled in [**1**] we need only take care of the locus of those bundles whose monodromy lies in the *normalizer of the one dimensional torus*. In this case it is easy to see that such a rank two bundle can be realised as a direct image of a line bundle from an unramified two sheeted cover $p : D \to C$ of C, and since we need the structure group to be $SL(2)$, the locus is precisely

$$\{p_*(L) | det(p_*(L)) \simeq \mathcal{O}_C\}.$$

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