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REDUCED GENUS-ONE GROMOV-WITTEN INVARIANTS

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Abstract

In a previous paper, we described a natural closed subset, $\overline{\mathfrak{M}}^0_{1,k}(X,A;J)$, of the moduli space $\overline{\mathfrak{M}}_{1,k}(X,A;J)$ of stable genus-one J-holomorphic maps into a symplectic manifold X. In this paper we generalize the definition of the main component to moduli spaces of perturbed, in a restricted way, J-holomorphic maps and conclude that $\overline{\mathfrak{M}}^0_{1,k}(X,A;J)$, just like $\overline{\mathfrak{M}}_{1,k}(X,A;J)$, carries a virtual fundamental class, which can be used to define symplectic invariants. These truly genus-one invariants constitute part of the standard genus-one Gromov-Witten invariants, which arise from the entire moduli space $\overline{\mathfrak{M}}_{1,k}(X,A;J)$. The new invariants are more geometric and can be used to compute the genus-one GW-invariants of complete intersections, as shown in a separate paper.

1. Introduction

1.1. Background and Motivation. Let (X,ω,J) be a compact almost Kähler manifold. In other words, (X,ω) is a symplectic manifold and J is an almost complex structure on X tamed by ω, i.e.

$$\omega(v,Jv)>0 \qquad \forall v\in TX-X.$$

If g,k are nonnegative integers and $A\in H_2(X;\mathbb{Z})$, let $\overline{\mathfrak{M}}_{g,k}(X,A;J)$ denote the moduli space of (equivalence classes of) stable J-holomorphic maps from genus-g Riemann surfaces with k marked points in the homology class A. Let

$$\mathfrak{M}^0_{g,k}(X,A;J)\subset\overline{\mathfrak{M}}_{g,k}(X,A;J)$$

be the subspace consisting of the stable maps $[\mathcal{C},u]$ such that the domain $\mathcal{C}$ is a smooth Riemann surface. The compact moduli space $\overline{\mathfrak{M}}_{g,k}(X,A;J)$ was constructed in order to "compactify" $\mathfrak{M}^0_{g,k}(X,A;J)$ and to define invariants of (X,ω) enumerating J-holomorphic curves of genus g in X. If $g=0$, $(X,\omega;A)$ is positive in a certain sense, and J is generic, then $\mathfrak{M}^0_{g,k}(X,A;J)$ is a dense open subset of $\overline{\mathfrak{M}}_{g,k}(X,A;J)$ and

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