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ON THE CONTACT CLASS IN HEEGAARD FLOER HOMOLOGY

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Abstract

We present an alternate description of the Ozsváth-Szabó contact class in Heegaard Floer homology. Using our contact class, we prove that if a contact structure (M, ξ) has an adapted open book decomposition whose page S is a once-punctured torus, then the monodromy is right-veering if and only if the contact structure is tight.

1. Introduction

In the paper [**OS5**], Ozsváth and Szabó defined an invariant of a contact 3-manifold (M, ξ) which lives in the Heegaard Floer homology $\widehat{HF}(-M)$ of the manifold M with reversed orientation. It is defined via the work of Giroux [**Gi2**], who showed that there is a 1-1 correspondence between isomorphism classes of open book decompositions modulo positive stabilization and isomorphism classes of contact structures on closed 3-manifolds. Ozsváth and Szabó associated an element in Heegaard Floer homology to an open book decomposition and showed that its homology class is independent of the choice of the open book compatible with the given contact structure. They also showed that this invariant $c(\xi)$ is zero if the contact structure is overtwisted, and that it is nonzero if the contact structure is symplectically fillable. The *contact class* $c(\xi)$ has proven to be extremely powerful at (i) proving the tightness of various contact structures and (ii) distinguishing tight contact structures, especially in the hands of Lisca-Stipsicz [**LS1**, **LS2**] and Ghiggini [**Gh**].

The goal of this paper is to introduce an alternate, more hands-on, description of the contact class in Heegaard Floer homology and to use it in the context of our program of relating right-veering diffeomorphisms to tight contact structures.

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