J. DIFFERENTIAL GEOMETRY
83 (2009) 189-212

CODIMENSION ONE FOLIATIONS WITH BOTT-MORSE SINGULARITIES I

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Abstract

In this article we show how the classical theory of Reeb and others extends to the case of codimension one singular foliations on closed oriented manifolds, provided that at the singular set $\mathrm{sing}(\mathcal{F})$, the foliation is locally defined by Bott-Morse functions which are transversely centers. We prove, in this setting, the equivalent of the local and the complete stability theorems of Reeb. We show that if $\mathcal{F}$ has a compact leaf with finite fundamental group, or if a component of $\mathrm{sing}(\mathcal{F})$ has codimension ≥ 3 and finite fundamental group, then all leaves of $\mathcal{F}$ are compact and diffeomorphic, $\mathrm{sing}(\mathcal{F})$ consists of two connected components, and there is a Bott-Morse function $f : M \to [0,1]$ such that $f\colon M \setminus \mathrm{sing}(\mathcal{F}) \to (0,1)$ is a fiber bundle defining $\mathcal{F}$ and $\mathrm{sing}(\mathcal{F}) = f^{-1}(\{0,1\})$. This yields a topological description of the type of leaves that appear in these foliations, and also the type of manifolds admitting such foliations. These results unify and generalize well known results for cohomogeneity one isometric actions, and a theorem of Reeb for foliations with Morse singularities of center type.

Introduction

Cohomogeneity one isometric actions of Lie groups, *i.e.*, actions where the principal orbits have codimension 1, play an important role in Differential Geometry, particularly in the Theory of Minimal Submanifolds (see for instance [**13**]). A basic well-known fact about these actions is that whenever the group and the manifold are compact, if all orbits are principal then the space of orbits is S^1, and if there are special orbits then there are exactly two of them and the space of orbits is the interval $[0,1]$. Notice that such an action defines a codimension one foliation with compact leaves and singular set the special orbits. Since the action is isometric, the intersection of the orbits with a slice Σ transverse to a special orbit corresponds to a Morse singularity of center type.

The second named author was partially supported by CONACYT and DGAPA-UNAM, México.

Received 06/26/06.