

CALABI-YAU COMPONENTS IN GENERAL TYPE HYPERSURFACES

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Abstract

For a one-parameter family $(V, \{\Omega_i\}_{i=1}^{p_g})$ of general type hypersurfaces with bases of holomorphic n -forms, we construct open covers $V = \bigcup_{i=1}^{p_g} U_i$ using tropical geometry. We show that after normalization, each Ω_i is approximately supported on a unique U_i and such a pair approximates a Calabi-Yau hypersurface together with its holomorphic n -form as the parameter becomes large. We also show that the Lagrangian fibers in the fibration constructed by Mikhalkin [9] are asymptotically special Lagrangian. As the holomorphic n -form plays an important role in mirror symmetry for Calabi-Yau manifolds, our results is a step toward understanding mirror symmetry for general type manifolds.

1. Introduction

Calabi-Yau manifolds are Kähler manifolds with zero first Chern class. By Yau's theorem [14], they admit Ricci flat Kähler metrics. They play important roles in String theory as internal spaces. Up to a scalar multiple, there exists a unique holomorphic volume form $\Omega \in H^{n,0}(Y)$ on any Calabi-Yau manifold Y . In the SYZ proposal [12] for the Mirror Symmetry conjecture, Strominger, Yau and Zaslow conjectured that mirror symmetry is a generalization of the Fourier-Mukai transformation along dual special Lagrangian torus fibrations on mirror Calabi-Yau manifolds and it is called the "SYZ transformation". Recall that a Lagrangian submanifold L in Y is called *special* if $\text{Im } \Omega|_L = 0$. It is not easy to construct special Lagrangian fibrations on Calabi-Yau manifolds. Nevertheless, Lagrangian fibrations do exist on Calabi-Yau hypersurfaces in \mathbb{CP}^{n+1} , or other toric varieties, by the work of Gross [7], Ruan [11] and others.

There are generalizations of the Mirror Symmetry conjecture for Fano manifolds (i.e. positive first Chern class) and also recently for general type manifolds (e.g. negative first Chern class). There are many Fano manifolds which are toric varieties and therefore they admit natural

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