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HAMILTONIAN STATIONARY SHRINKERS AND EXPANDERS FOR LAGRANGIAN MEAN CURVATURE FLOWS

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Abstract

We construct examples of shrinkers and expanders for Lagrangian mean curvature flows. These examples are Hamiltonian stationary and asymptotic to the union of two Hamiltonian stationary cones found by Schoen and Wolfson in [**SWO**]. The Schoen-Wolfson cones $C_{p,q}$ are obstructions to the existence problems of special Lagrangians or Lagrangian minimal surfaces in the variational approach. It is known that these cone singularities cannot be resolved by any smooth oriented Lagrangian submanifolds. The shrinkers and expanders that we found can be glued together to yield solutions of the Brakke motion-a weak formulation of the mean curvature flow. For any coprime pair (p, q) with $p > q > 1$, we construct such a solution that resolves one single Schoen-Wolfson cone $C_{p,q}$. Note that $C_{p,q}$ is stable only if $p - q = 1$. It thus provides an evidence to Schoen-Wolfson's conjecture that the $(2, 1)$ cone is the only area-minimizing cone. Higher dimensional generalizations are also obtained.

1. Introduction

The existence of special Lagrangians in Calabi-Yau manifolds received much attention recently due to the critical role it plays in the T-duality formulation of Mirror symmetry of Strominger-Yau-Zaslow [**SYZ**]. Schoen and Wolfson took up the variational approach of constructing special Lagrangians by minimizing volumes in suitable Lagrangian classes. They discovered non-flat Lagrangian cones that are Hamiltonian stationary [**SWO**]. The existence of special Lagrangians can be established once these cone singularities are excluded. However, these singularities do occur in the Lagrangian minimizers in some K-3 surfaces, see [**WO**]. Another potential approach to the construction of special Lagrangians is the mean curvature flow- as the negative gradient flow of the volume functional. However, the long-time existence of

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