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DYNAMICS OF AUTOMORPHISMS ON PROJECTIVE COMPLEX MANIFOLDS

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Abstract

We show that the dynamics of automorphisms on all projective complex manifolds X (of dimension 3, or of any dimension but assuming the Good Minimal Model Program or Mori's Program) are canonically built up from the dynamics on just three types of projective complex manifolds: complex tori, weak Calabi-Yau manifolds and rationally connected manifolds. As a by-product, we confirm the conjecture of Guedj [**20**] for automorphisms on 3-dimensional projective manifolds, and also determine $\pi_1(X)$.

1. Introduction

We work over the field $\mathbb{C}$ of complex numbers.

We show that the dynamics of automorphisms on all projective complex manifolds (of dimension 3, or of any dimension but assuming the Good Minimal Model Program or Mori's Program) are canonically built up from the dynamics on just three types of projective complex manifolds: complex tori, weak Calabi-Yau manifolds, and rationally connected manifolds.

For a similar phenomenon on the dynamics in dimension 2, we refer to [**6**]. Here a projective manifold X is *weak Calabi-Yau* or simply *wCY* if the Kodaira dimension $\kappa(X) = 0$ and the first Betti number $B_1(X) = 0$. A projective manifold X is *rationally connected* (the higher dimensional analogue of a rational surface) if any two points on X are connected by a rational curve; see [**5**] and [**31**]. For a smooth projective surface X, it is wCY if and only if X itself or its étale double cover is birational to a K3 surface, while X is rationally connected if and only if it is a rational surface.

For the recent development on complex dynamics, we refer to the survey articles [**13**] and [**58**] and the references therein. See also [**7**], [**11**], [**37**], [**42**] and [**55**]. For algebro-geometric approach to dynamics of automorphisms due to Oguiso, see [**47**], [**46**] and [**45**].

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