

ON THE GENUS-ONE GROMOV-WITTEN INVARIANTS OF COMPLETE INTERSECTIONS

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Abstract

We state and prove a long-elusive relation between genus-one Gromov-Witten of a complete intersection and twisted Gromov-Witten invariants of the ambient projective space. As shown in a previous paper, certain naturally arising cones of holomorphic vector bundle sections over the main component $\overline{\mathfrak{M}}_{1,k}^0(\mathbb{P}^n, d)$ of the moduli space of stable genus-one holomorphic maps into \mathbb{P}^n have a well-defined euler class. In this paper, we extend this result to moduli spaces of perturbed, in a restricted way, J -holomorphic maps. This extension is used to show that these cones are the correct genus-one analogues of the vector bundles relating genus-zero Gromov-Witten invariants of a complete intersection to those of the ambient projective space. A relationship for higher-genus invariants is conjectured as well.

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