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HIGHER ORDER BERGMAN FUNCTIONS AND EXPLICIT CONSTRUCTION OF MODULI SPACE FOR COMPLETE REINHARDT DOMAINS

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Abstract

In this article we introduce higher order Bergman functions for bounded complete Reinhardt domains in a variety with possibly isolated singularities. These Bergman functions are invariant under biholomorhic maps. We use Bergman functions to determine all the biholomorhic maps between two such domains. As a result, we can construct an infinite family of numerical invariants from the Bergman functions for such domains in A_n variety $\{(x, y, z) \in \mathbb{C}^3 : xy = z^{n+1}\}$. These infinite family of numerical invariants are actually a complete set of invariants for either the set of all bounded strictly pseudoconvex complete Reinhardt domain in A_n variety or the set of all bounded pseudoconvex complete Reinhardt domains with real analytic boundaries in A_n variety. In particular the moduli space of these domains in A_n variety is constructed explicitly as the image of this complete family of numerical invariants. It is well known that A_n variety is the quotient of cyclic group of order $n+1$ on $\mathbb{C}^2$. We prove that the moduli space of bounded complete Reinhardt domains in A_n variety coincides with the moduli space of the corresponding bounded complete Reinhardt domains in $\mathbb{C}^2$. Since our complete family of numerical invariants are computable, we have solved the biholomorphically equivalent problem for large family of domains in $\mathbb{C}^2$.

1. Introduction

Let D_1 and D_2 be two domains in $\mathbb{C}^n$. One of the most fundamental problems in complex geometry is to determine conditions which will imply that D_1 and D_2 are biholomorphically equivalent. For $n = 1$, the celebrated Riemann mapping theorem states that any simply connected domains in $\mathbb{C}$ are biholomorphically equivalent. For $n \geqslant 2$, it is well known that there are lots of domains which are topologically equivalent to the ball but not necessarily biholomorphically equivalent

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