

## DIRAC GEOMETRY, QUASI-POISSON ACTIONS AND $D/G$ -VALUED MOMENT MAPS

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### Abstract

We study Dirac structures associated with Manin pairs  $(\mathfrak{d}, \mathfrak{g})$  and give a Dirac geometric approach to Hamiltonian spaces with  $D/G$ -valued moment maps, originally introduced by Alekseev and Kosmann-Schwarzbach [3] in terms of quasi-Poisson structures. We explain how these two distinct frameworks are related to each other, proving that they lead to isomorphic categories of Hamiltonian spaces. We stress the connection between the viewpoint of Dirac geometry and equivariant differential forms. The paper discusses various examples, including  $q$ -Hamiltonian spaces and Poisson-Lie group actions, explaining how presymplectic groupoids are related to the notion of “double” in each context.

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