

**EXPLICIT CONSTRUCTION OF NEW MOISHEZON
TWISTOR SPACES**

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Abstract

In this paper we explicitly construct Moishezon twistor spaces on $n\mathbf{CP}^2$ for arbitrary $n \geq 2$ which admit a holomorphic \mathbf{C}^* -action. When $n = 2$, they coincide with Y. Poon's twistor spaces. When $n = 3$, they coincide with the ones studied by the author in [14]. When $n \geq 4$, they are new twistor spaces, to the best of the author's knowledge. By investigating the anticanonical system, we show that our twistor spaces are bimeromorphic to conic bundles over certain rational surfaces. The latter surfaces can be regarded as orbit spaces of the \mathbf{C}^* -action on the twistor spaces. Namely they are minitwistor spaces. We explicitly determine their defining equations in \mathbf{CP}^4 . It turns out that the structure of the minitwistor space is independent of n . Further, we explicitly construct a \mathbf{CP}^2 -bundle over the resolution of this surface, and provide an explicit defining equation of the conic bundles. It shows that the number of irreducible components of the discriminant locus for the conic bundles increases as n does. Thus our twistor spaces have a lot of similarities with the famous LeBrun twistor spaces, where the minitwistor space $\mathbf{CP}^1 \times \mathbf{CP}^1$ in LeBrun's case is replaced by our minitwistor spaces found in [15].

1. Introduction

More than 15 years have passed since C. LeBrun [22] discovered a series of self-dual metrics and their twistor spaces, on the connected sum of complex projective planes. Basically they are obtained as a 1-dimensional reduction of the self-duality equation for conformal classes [1], and can be regarded as a hyperbolic version of gravitational multi-instantons by G. Gibbons-S. Hawking [4] and their twistor spaces by N. Hitchin [5]. Characteristic property of LeBrun's result is that it is completely explicit: for the twistor spaces, a bimeromorphic projective model is explicitly given by a defining equation and then bimeromorphic transformations are concretely given which produce actual twistor

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