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STRONG UNIQUENESS OF THE RICCI FLOW

Bing-Long Chen

Abstract

In this paper, we derive some local a priori estimates for the Ricci flow. This gives rise to some strong uniqueness theorems. As a corollary, let $g(t)$ be a smooth complete solution to the Ricci flow on $\mathbb{R}^3$, with the canonical Euclidean metric E as initial data, then $g(t)$ is trivial, i.e. $g(t) \equiv E$.

1. Introduction

The Ricci flow $\frac{\partial}{\partial t} g_{ij}(x,t) = -2R_{ij}(x,t)$, was introduced by Hamilton in [**7**]. The major application of this equation to lower dimensional topology has had a great impact in modern mathematics (see [**7**], [**8**], [**9**] [**12**], [**13**]). The power of these geometric applications grew out of the fundamental PDE theory of the equation. These two aspects had been intertwined all the time since the foundation of the Ricci flow.

In this paper, we go back to some fundamental PDE problems of this equation.

Let's look at one heuristic analogue, the standard heat equation $\left(\frac{\partial}{\partial t} - \triangle\right) u = 0$ on $\mathbb{R}^n$. If u grows slower than the function $e^{a|x|^2}$ for some $a > 0$, then u is unique for all such solutions with the same initial data. Moreover, if $|u| \mid_{t=0} \leq Ce^{a|x|^2}$, it is not hard to see the short time existence (of solutions of same type) from the heat kernel convolution. For the Ricci flow, the Ricci curvature behaves like twice derivative of logarithmic of the metric. So bounded curvature condition for the Ricci flow resembles growth $e^{a|x|^2}$ for the standard heat equation. Actually, the fundamental work [**14**] showed that on complete manifolds with bounded curvature, the Ricci flow always admits short time solutions of bounded curvature. X.P. Zhu and the author recently [**3**] proved that the uniqueness theorem holds for solutions in the class of bounded curvature. For an interesting application of this theorem to the theory of Ricci flow with surgery, we refer the readers to see [**4**] or relevant discussions in [**1**][**10**][**11**].

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