

**TAU-FUNCTIONS ON SPACES OF ABELIAN
DIFFERENTIALS AND HIGHER GENUS
GENERALIZATIONS OF RAY-SINGER FORMULA**

ALEKSEY KOKOTOV & DMITRY KOROTKIN

Abstract

Let w be an Abelian differential on a compact Riemann surface of genus $g \geq 1$. Then $|w|^2$ defines a flat metric with conical singularities and trivial holonomy on the Riemann surface. We obtain an explicit holomorphic factorization formula for the ζ -regularized determinant of the Laplacian in the metric $|w|^2$, generalizing the classical Ray-Singer result in $g = 1$.

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The work of AK was supported by NSERC. The work of DK was partially supported by Concordia Research Chair grant, NSERC, NATEQ and Humboldt foundation. We both thank Max-Planck-Institut für Mathematik at Bonn where the main part of this work was completed for support, hospitality and excellent working conditions.

Received 09/13/2005.