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POINTS IN PROJECTIVE SPACES AND APPLICATIONS

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Abstract

We prove the factoriality of a nodal hypersurface in $\mathbb{P}^4$ of degree d that has at most $2(d-1)^2/3$ singular points, and we prove the factoriality of a double cover of $\mathbb{P}^3$ branched over a nodal surface of degree $2r$ having less than $(2r-1)r$ singular points.

1. Introduction

Let Σ be a finite subset in $\mathbb{P}^n$ and $\xi \in \mathbb{N}$, where $n \geqslant 2$. Then the points of the set Σ impose independent linear conditions on homogeneous forms of degree ξ if and only if for every point $P \in \Sigma$ there is a homogeneous form of degree ξ that vanishes at every point of the set $\Sigma \setminus P$, and does not vanish at the point P. The latter is equivalent to the equality

$$h^1\Big(\mathcal{I}_\Sigma \otimes \mathcal{O}_{\mathbb{P}^n}(\xi)\Big) = 0,$$

where $\mathcal{I}_\Sigma$ is the ideal sheaf of the subset $\Sigma \subset \mathbb{P}^n$.

In this paper we prove the following result (see Section 2).

Theorem 1. *Suppose that there is a natural number $\lambda \geqslant 2$ such that at most λk points of the set Σ lie on a curve in $\mathbb{P}^n$ of degree k. Then*

$$h^1\Big(\mathcal{I}_\Sigma \otimes \mathcal{O}_{\mathbb{P}^n}(\xi)\Big) = 0$$

in the case when one of the following conditions holds:

- $\xi = \lfloor 3\lambda/2 - 3 \rfloor$ *and* $|\Sigma| < \lambda \lceil \lambda/2 \rceil$;
- $\xi = \lfloor 3\mu - 3 \rfloor$, $|\Sigma| \leqslant \lambda\mu$ *and* $\lfloor 3\mu \rfloor - \mu - 2 \geqslant \lambda \geqslant \mu$ *for some* $\mu \in \mathbb{Q}$;
- $\xi = \lfloor n\mu \rfloor$, $|\Sigma| \leqslant \lambda\mu$ *and* $(n-1)\mu \geqslant \lambda$ *for some* $\mu \in \mathbb{Q}$.

Let us consider applications of Theorem 1.

Definition 2. An algebraic variety X is factorial if $\mathrm{Cl}(X) = \mathrm{Pic}(X)$.

We assume that all varieties are projective, normal, and defined over $\mathbb{C}$.

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