

A REGULARITY AND COMPACTNESS THEORY FOR IMMERSED STABLE MINIMAL HYPERSURFACES OF MULTIPLICITY AT MOST 2

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Abstract

We prove that a stable minimal hypersurface of an open ball which is immersed away from a closed (singular) set of finite co-dimension 2 Hausdorff measure and weakly close to a multiplicity 2 hyperplane must in the interior be the graph over the hyperplane of a 2-valued function satisfying a local $C^{1,\alpha}$ estimate. This regularity is optimal under our hypotheses. As a consequence, we also establish compactness of the class of stable minimal hypersurfaces of an open ball which have volume density ratios uniformly bounded by $3 - \delta$ for any fixed $\delta \in (0, 1)$ and interior singular sets of vanishing co-dimension 2 Hausdorff measure.

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1. Introduction

Our goal in this paper is to study the local structure of immersed, possibly branched, stable minimal hypersurfaces of the $(n + 1)$ -dimensional Euclidean space for arbitrary $n \geq 2$. Assuming the singular set of such

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