J. DIFFERENTIAL GEOMETRY
80 (2008) 1-22

MINIMAL LAGRANGIAN DIFFEOMORPHISMS BETWEEN DOMAINS IN THE HYPERBOLIC PLANE

Simon Brendle

Abstract

Let N be a complete, simply-connected surface of constant curvature $\kappa \leq 0$. Moreover, suppose that Ω and $\tilde{\Omega}$ are strictly convex domains in N with the same area. We show that there exists an area-preserving diffeomorphism from Ω to $\tilde{\Omega}$ whose graph is a minimal submanifold of $N \times N$.

1. Introduction

This paper is concerned with the boundary regularity of minimal Lagrangian diffeomorphisms. The notion of a minimal Lagrangian diffeomorphism was introduced by R. Schoen. In [**7**], Schoen proved an existence and uniqueness result for minimal Lagrangian diffeomorphisms between hyperbolic surfaces:

Theorem 1.1 (R. Schoen [**7**])**.** *Let N be a compact surface of genus greater than 1, and let $g, \tilde{g}$ be a pair of hyperbolic metrics on N. Then there exists a unique diffeomorphism $f : N \to N$ with the following properties:*

(i) *f is area-preserving.*
(ii) *f is homotopic to the identity.*
(iii) *The graph of f is a minimal submanifold of $(N, g) \times (N, \tilde{g})$.*

Theorem 1.1 was subsequently generalized by Y.I. Lee [**5**]. M.T. Wang [**10**] gave an alternative proof of the existence part of Theorem 1.1 using mean curvature flow.

In this paper, we study an analogous problem for surfaces with boundary. Throughout this paper, we will assume that N is a complete, simply-connected surface of constant curvature $\kappa \leq 0$. Suppose that Ω and $\tilde{\Omega}$ are domains in N with smooth boundary, and let f be a diffeomorphism from Ω to $\tilde{\Omega}$. We will say that f is a minimal Lagrangian diffeomorphism if the following conditions are satisfied:

(i) f is area-preserving.
(ii) f is orientation-preserving.
(iii) The graph of f is a minimal submanifold of $N \times N$.

Received 11/06/2006.