

# VANISHING THEOREMS ON COMPLETE KÄHLER MANIFOLDS AND THEIR APPLICATIONS

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## 1. Introduction

Semi-positive line bundles over compact Kähler manifolds have been the focus of studies for decades. Among them, there are several straddling vanishing theorems such as the Kodaira-Nakano Vanishing Theorem, Vesentini-Gigante-Girbau Vanishing Theorems and Kawamata-Viehweg Vanishing Theorem. As a corollary of the above mentioned vanishing theorems one can easily show that a line bundle over compact Kähler manifolds with negative degree has no non-trivial holomorphic sections. The high cohomology vanishing theorems for non-compact complex manifolds were also studied by several authors. Among them, there are the Nakano's vanishing theorem for Nakano-positive vector bundle over weakly 1-complete manifolds, and Andreotti-Vesentini's vanishing theorem for the  $q$ -complete manifolds. In the case where  $M$  is a non-compact manifold there are also many works on the finiteness of cohomology group. One of these results proved by N. Mok in [16] gave the finite dimensional estimate for the space of  $L^2$ -sections in the case where  $M$  is a complete noncompact Kähler manifold with finite volume.

In this paper we first show some vanishing theorems for the  $L^2$ -sections of the holomorphic vector bundles over complete nonparabolic Kähler manifolds. By applying the vanishing results and the  $L^2$ -estimate of  $\bar{\partial}$  of Andreotti-Vesentini, we show, among other things, that if  $M$  is a non-parabolic Kähler manifold with nonnegative Ricci curvature and

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