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CORRESPONDENCE OF HERMITIAN MODULAR FORMS TO CYCLES ASSOCIATED TO $SU(p, 2)$

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In the previous papers [11], [12] we have given a correspondence, in the form of a lifting through a theta function, from Hermitian modular forms of degree r to codimension r geodesic cycles in the locally symmetric spaces associated to $SU(p, 1)$. Our purpose here is to extend this correspondence to the case where the target is associated to a unitary group of rank greater than one: $SU(p, 2)$. In [11, §1] we have defined geodesic cycles of codimension $2r$, $1 \leqslant r \leqslant p - 1$, associated to $SU(p, 2)$. In this paper we make the further restriction that $r = 1$. This has the merit that while exhibiting some of the new phenomena in higher rank, it is basically differential forms of degree $(2, 2)$ we are dealing with and some steps are manageable by direct calculations.

The theory of Weil representation and theta functions for dual reductive pairs has been utilized to construct liftings of automorphic forms in abundant cases (cf. the references to [5], [8], [11], [12]). All these give correspondence of automorphic forms associated to two different groups. Other than some accidental lower dimensional cases it is considerably more surprising, and technically more subtle, that this machinery embodies a lift from automorphic forms to harmonic forms dual to special cycles. The technical subtleties appear inevitable since one is trying to link automorphic objects with higher dimensional geometric objects.

As first found for $SO(p, 1)$ in [8], and then a modified version found for $SU(p, 1)$ in [11], [12], this link to geometry comes from two ingredients.

(i) *A construction of the harmonic form dual to such a cycle as a special value via analytic continuation of a one (complex) parameter family of dual forms.* Let $\mathfrak{D}$ be the bounded symmetric domain for $G = SU(p, 2)$ and Γ a cocompact discrete subgroup of G. In §1 we consider a particular subdomain $\mathfrak{D}_1 \subset \mathfrak{D}$ of codimension 2. Denote by $G_1 \subset G$ the subgroup leaving $\mathfrak{D}_1$ invariant and

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