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# DEFORMING METRICS IN THE DIRECTION OF THEIR RICCI TENSORS

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*(An appendix to a paper of R. Hamilton)*

**1. Introduction.** In [2], R. Hamilton has proved that if a compact manifold $M$ of dimension three admits a $C^\infty$ Riemannian metric $g_0$ with positive Ricci curvature, then it also admits a metric $\bar{g}$ with constant (positive) sectional curvature, and is thus (a quotient of) the sphere $S^3$. In fact, he shows that the original metric can be deformed into the constant-curvature metric by requiring that, for $t \geqslant 0$, $x \in M$ and $g = g(t, x)$,

$$(1) \qquad \frac{\partial g}{\partial t} = \frac{2}{3} r_t g - 2\mathrm{Ric}(g), \qquad g(0, x) = g_0(x),$$

where $\mathrm{Ric}(g)$ is the Ricci curvature of $g$ on $M$ at time $t$, and $r_t$ is the average scalar curvature of the metric $g_t = g(t, x)$ over $M$, i.e.,

$$r_t = \frac{1}{\mathrm{Vol}_{g_t}(M)} \int_M \mathrm{Scal}(g_t)\, dV_{g_t}.$$

Hamilton's proof has two parts. In the first part, he proves local-in-time existence for the initial-value problem (IVP) (1), which is equivalent to proving local existence for the IVP

$$(2) \qquad \frac{\partial g}{\partial t} = -2\mathrm{Ric}(g), \qquad g(0, x) = g_0(x)$$

(see [2, §3]). This part of the proof is valid for all dimensions $n \geqslant 3$. In the second part, which is specific to three dimensions, he proves that, as $t$ approaches $\infty$, $g(t, x)$ approaches $\bar{g}(x)$ and that the Ricci curvature of $g$ remains positive throughout the deformation.

To do the first (local) part of the proof, Hamilton uses a deep and powerful theorem from analysis: the Nash-Moser implicit-function theorem. (Some

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