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THE TOPOLOGY OF CERTAIN RIEMANNIAN MANIFOLDS WITH POSITIVE RICCI CURVATURE

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1. Let M be a complete connected Riemannian manifold of dimension n, and let Ric denote its Ricci curvature. Understanding the Ricci curvature is one of the important problems in today's geometry. In these notes, we assume that $\mathrm{Ric} \geqslant n-1$. The classical theorem of Myers then asserts that M is compact and has diameter $d_M \leqslant \pi$. R. Bishop showed that the volume of M also satisfied $\mathrm{vol}_M \leqslant \mathrm{vol}_{S^n}$, where S^n is the unit Euclidean sphere in $\mathbf{R}^{n+1}$, and that the equality holds only if M is isometric to S^n. In [3], S. Y. Cheng proves

Theorem A. *If $d_M = \pi$, then M is isometric to S^n.*

It is interesting to ask to what extent these theorems can be perturbed. Our main result is

Main Theorem. *Given any upper bound κ for the sectional curvature of M, there exists a constant $v > 0$, depending only on n and κ, such that whenever $\mathrm{vol}_M \geqslant (1-v)\mathrm{vol}_{S^n}$, then M has the homotopy type of S^n.*

By using some of the same methods, we can also show

Theorem B. *There is a constant $\rho > 0$, depending only on n, such that if M has the injectivity radius $i_M > \pi - \rho$, then M is homeomorphic to S^n.*

In §2 of these notes, we describe the main tools which can be used to prove these theorems. In §§3 and 4, we outline the proofs of Theorem B and Main Theorem. In §5, we describe a new geometric proof for Theorem A. Finally, we discuss some remarks and open question in §6. Details and additional applications will appear in [10]. The author would like to express gratitude to D. Gromoll for many helpful discussions.

2. Our main tool is the following observation in [7], based on an earlier work by Bishop. We denote by $B(r; p)$ the open metric ball of radius r and center p in M, and let $\hat{B}(r)$ be an open ball in S^n of radius r. Then we have

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