A NOTE ON A COUNTEREXAMPLE
OF DELGADO

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In this note we correct some incorrect analysis appearing in the paper of J. A. Delgado [1].

The example concerns two plane curves γ_1, γ_2, which both are regular and complete, and have nonnegative curvature κ, i.e., $\kappa(\gamma_1) \geqslant 0$, $\kappa(\gamma_2) \geqslant 0$.

In this example Delgado intended to show that γ_1 and γ_2 are internally tangent at 0 and that $\kappa(\gamma_1(t)) \geqslant \kappa(\gamma_2(s))$ whenever $N_1(t) = N_2(s)$ where N_1 (resp. N_2) is the unit outward normal of γ_1 (resp. γ_2). He also showed that γ_1 is not contained in the convex region formed by γ_2, thus showing that Blaschke's theorem does not apply to curves with nonnegative rather than positive curvature. However his analysis is incorrect. The example should go as follows:

$$\gamma_1(t) = \left(pt, t^4 \right), \quad t \in \mathbf{R}, \quad p > 1,$$

$$\gamma_2(s) = \begin{cases} \left(s, (s-1)^4 \right), & s \in \mathbf{R}, \quad s \geqslant 1, \\ (s, 0), & s \in \mathbf{R}, \quad |s| \leqslant 1, \\ \left(s, (s+1)^4 \right), & s \in \mathbf{R}, \quad s \leqslant -1, \end{cases}$$

$$N_1(t) = \frac{1}{\left(p^2 + 16t^2 \right)^{1/2}} \left(4t^3, -p \right),$$

$$N_2(s) = \begin{cases} \dfrac{1}{\left(1 + 16(s-1)^6 \right)^{1/2}} \left(4(s-1)^3, -1 \right), & \text{if } s \geqslant 1, \\ (0, -1), & \text{if } |s| \leqslant 1, \\ \dfrac{1}{\left(1 + 16(s+1)^6 \right)^{1/2}} \left(4(s+1)^3, -1 \right), & \text{if } s \leqslant -1. \end{cases}$$