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MINIMUM IMBEDDINGS OF COMPACT SYMMETRIC SPACES OF RANK ONE

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1. Introduction

Let M be a compact differentiable manifold of dimension n, and

$$\varphi : M \to R^{n+k} \tag{1.1}$$

an immersion of M into a Euclidean space R^{n+k} of dimension $n + k$. The total curvature, in the sense of Chern and Lashof [1], [12], can be defined as follows.

Let B be the set of unit normal vectors of M in R^{n+k}. Then B is a bundle of $(k-1)$-sphere over M and is a manifold of dimension $n + k - 1$. Let S be the unit $(n + k - 1)$-sphere of R^{n+k}, $d\sigma$ the volume element of S, and

$$c_{n+k-1} = \int_S d\sigma \tag{1.2}$$

the volume of S. If

$$\nu : B \to S \tag{1.3}$$

is the Gauss map, which assigns each unit normal vector of B the unit vector through the origin and parallel to the normal vector, then the total curvature of the immersed manifold M is defined as

$$\frac{1}{c_{n+k-1}} \int_B |\nu^* d\sigma| \, . \tag{1.4}$$

Since the total curvature depends on M as well as $\varphi : M \to R^{n+k}$, we shall denote it by $\tau(M, \varphi, R^{n+k})$ or simply by $\tau(\varphi)$.

The height function h_a in the direction $a \in R^{n+k}$ takes the value

$$h_a(x) = (a, \varphi(x)) \tag{1.5}$$

at $x \in M$, where (,) denotes the usual inner product on R^{n+k}. $x \in M$ is a

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