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PERFECT TENSORS ON A MANIFOLD

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Introduction

In one of a series of papers on the holonomy group, Hlavaty [2] calls the infinitesimal holonomy group of a connection *perfect* if its Lie algebra can be found from the curvature tensor alone, that is, the covariant derivatives of the curvature tensor will add nothing new to this Lie algebra. In [4] we generalized this definition to an arbitrary tensor and gave a necessary and sufficient condition that a tensor be perfect. This condition was expressed in terms of restrictions on a certain set of local tensor fields. In this paper, we call a set of tensor fields perfect if they satisfy this condition in every coordinate neighborhood.

Our perfect set of tensor fields reduces to several well-known concepts in special cases. For example, if there is only one tensor field in a perfect set, this tensor field is recurrent (or covariant constant). Also, if the tensor fields in a perfect set are all vector fields, then their linear span at all points of the manifold form a parallel field of planes (see Walker [5]).

To a given tensor field on a manifold M, Chern [1] has associated a set of functions on the bundle of frames over M. Wong [7] has given a necessary and sufficient condition on these functions such that the associated tensor is covariant constant or recurrent. He also gives [8] a necessary and sufficient condition on these functions so that there exists a connection on the manifold M, with respect to which the associated tensor is covariant constant or recurrent. The present paper extends Wong's results to a perfect set of tensors. In this regard, Wong's theorems [7, Theorem 3.9], [8, Theorem 1.2] are special cases of Theorems 2.4 and 2.8. Using our characterization of perfect tensor fields, in §5 we are able to prove a fundamental result on fields of planes, namely, *every field of planes on M is parallel with respect to some connection on M.*

In §3, we examine the set of covectors that occur in the definition of a perfect set of tensors. A necessary and sufficient condition is obtained guaranteeing that the recurrence covector of a recurrent tensor is locally a gradient. This is then generalized to the set of covectors mentioned above. §4 is devoted to applications of the previous results.

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